

1. Prove that for all  $\alpha$  :

$$2 \sin\left(\frac{\pi}{4} - \alpha\right) \cos\left(\frac{\pi}{4} + \alpha\right) = 1 - \sin \alpha$$

2. Prove that

$$\frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} = \tan \alpha - \cot \alpha$$

3. Prove that

$$\sin x \cos x - \sin x - \cos x \geq -1$$

4. Prove that

$$\cos^2\left(\frac{\pi}{4} - \alpha\right) - \sin\left(\frac{2\pi}{3} - \alpha\right) \cdot \sin\left(\alpha - \frac{\pi}{3}\right) = \frac{3}{4}$$

5. For what real numbers is it true that

$$\sin x = \frac{\sin^2 x - \cos^2 x}{\cos 2x} \cot 2x$$

6. For what real numbers is the following equality true?

$$\tan x \cot x = \sin x \cos x$$

7. Is it true for all rational numbers  $x$  that

$$\frac{1 - 2 \sin^2 x}{\cos 2x} = 1$$

8. For what real numbers is the following inequality true?

$$\frac{1}{\sin^2} - \frac{2}{\sin x} \leq \cot^2 - 1$$

9. The length of the hypotenuse of an isosceles right-angled triangle is 2m. The right angle is divided into  $15^\circ$  angles by five straight lines. What are the lengths of the segments of these lines within the triangle?
10. The sum of the two sides of a right-angled triangle is two-thirds of its hypotenuse. The area of the triangle is  $10 \text{ cm}^2$ . What are the side and the angles of the triangle?
11. The area of a symmetric trapezoid is  $144 \text{ m}^2$ . The lengths of the two parallel sides are 5 m and 30 dm. What are the angles of this trapezoid?
12. The length of the base of an isosceles triangle is 10cm. The altitudes drawn to the side arms are 6 cm long. What is the radius of the circle inscribed?