

1. Graph the following functions!

$$f(x) = \log_3(x - 1) - 2, \quad g(x) = \log_{\frac{1}{2}}(4 - x) + 4$$

2. Calculate the value of these expressions!

$$10^{\log_{100} 5 - \log_3 27},$$

$$3^{\log_{27} 5 - \log_{\frac{1}{9}} 6}$$

3. Solve the equations!

$$\lg(x + 4) - \lg 4 = \lg(2x - 11),$$

$$\lg(7x^2 - 8) - \lg(7x - 12) = 1,$$

$$\log_2 x + \log_4 x + \log_8 x = 11,$$

$$\frac{1}{2} \log_3(x + 1) - \log_3 \sqrt{x + 4} = \log_3 4, 5 - 2,$$

$$\frac{\lg(3x-5)}{\lg(2x-3)} = 1,$$

$$\lg \sqrt{3x - 2} + \lg 4x - 7 = \lg 2,$$

$$4^{x+\frac{1}{2}} + 31 \cdot 2^{x-1} = 4,$$

$$2^{x-2} + 8^{\frac{x}{3}-1} - 4^{0,5x-2} = 10$$

4. Find the domain of these expressions!

$$\log_{x-1} \frac{4}{5-x},$$

$$\log_{2x}(x^2 - x - 2),$$

$$\log_{3-x} \frac{-4}{5+x},$$

$$\log_5 \frac{3+x}{2-x}$$

5. Solve the following system of equations!

1) $\log_2(2x + y) - \log_2(2x - y) = 1$

2) $\log_3(2x + y) + \log_3(2x - y) = 1$

1) $2^x + 5 \cdot 7^y = 40$

2) $2^x - 3 \cdot 7^y = -1$

1) $\lg x - \lg y = 2$

2) $x - 10y = 900$

1) $\lg x - 2 \cdot \lg y = 3$

2) $5 \cdot \lg x - \lg y = 4$

7. When comparing statistical information obtained from various countries, social scientists are using the following empirical formula:

$$E = 75 - 5 \cdot 10^{\frac{6000-G}{6090}}$$

In the formula E is the mean life expectancy in years at the time of birth, G is the gross domestic product (GDP) expressed in terms of the value of the US dollar in 1980. Find the difference in the GDP if the mean life expectancy grow from 40 years to 50.

8. The value of the car C can be represented by the formula $C = 1000 + 17000e^{-\frac{t}{3}}$, where t is the age in years from new.

Calculate the price of the new car!

Calculate the age of the car when its value first falls below £3000.

9. The number of seals on an island can be represented by the equation

$$P = 200 - 120e^{-t/10}$$

where P is the number of seals on the island and t is the time in years after January 1st 2005.

Find the number of seals on the island on January 1st 2005.

Calculate the number of extra seals on the island by January 1st 2010.

Find the year when the population rise over 180.

10. A technician measured the amount of radio-active substance as 10 mg. Ten hours later, the another measurement showed that there were 3 mg remaining. What is the half -life of these substance? (exponential decay equation: $A = A_0 \cdot (0,5)^{\frac{t}{T}}$, where t is the half-life, A_0 is the original amount, A is the amount of radio-active material after t years.

11. A scientist measures the amount of radio-active material as 15 mg. Three hours later a second measurement shows that the amount of the material decreased by 10%. What is the half-life of this radio-active material?